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# Cyclic Behaviour of Saturated Sands of Kalpakkam Region, Tamil Nadu

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## Introduction

Site-specific nature of damage to built environment has been demonstrated by many large earthquakes in different parts of the world. Severity of the damage caused by the 2001 Bhuj earthquake, not only in the epicentral regions, but also in cities such as Ahmedabad as far as 200 km away from the source, drew the attention of engineers and scientists in India. The need for more systematic and professional approach in earthquake geotechnical studies is recognized. The important information required in the engineering design of new and the retrofit of existing structures considering the earthquake forces is the site-specific dynamic soil properties. The dynamic soil properties required as inputs for ground response analysis include the shear wave velocity, the variation of normalized shear modulus and material damping with shear strain.

The behaviour of soil under cyclic loading is nonlinear and depends on several factors including cyclic shear strain amplitude, number of loading cycles, soil type and mean principal effective stress. Nonlinear hysterestic soil behaviour is approximated by equivalent linear analysis, where soil constitutive behaviour is defined by the secant shear modulus (G), Poisson's ratio ($\upsilon$), and damping ratio ($\xi$) (Seed and Idriss 1970; Hardin and Drnevich; 1972, Vucetic and Dodry 1991). The secant shear modulus of soil varies with cyclic shear strain amplitude ($\gamma$). At low strain amplitudes, the secant shear modulus is high and is known as maximum shear modulus ($G_{max}$), but it decreases as the strain amplitude increases. The secant shear modulus normalized by maximum shear modulus is known as modulus ratio ($G/G_{max}$). The variation of modulus ratio with cyclic shear strain is described graphically by modulus reduction curve (Kramer 1996). Damping which is a measure of energy dissipation in a loading cycle, increases with increasing magnitude of cyclic shear strain. The graphical representation of the damping ratio ($\xi$) versus the cyclic shear strain is known as damping ratio curve in literature.

The modulus reduction and the damping ratio curves most often used for cohesionless soils, such as sands and gravels, are those proposed by Seed and

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Idriss (1970). These modulus reduction and damping ratio curves are independent of the number of loading cycles as well as the relative density, the sand type, and the confining stress. However, the modulus reduction and the damping ratio of low plasticity soils are more influenced by the effective confining stress (Iwasaki et al. 1978; Kokusho 1980; Ishibashi and Zhang 1993).

The objective of this study is to assess the cyclic behaviour of saturated sand samples collected from the nuclear power plant site, Kalpakkam, located at south-east coast of India. Strain-controlled undrained cyclic triaxial tests are conducted on saturated sand to evaluate the shear modulus and damping at medium and high strain levels. The estimation of shear modulus at very low strain levels are made based on shear-wave velocity measurements by using piezoelectric bender elements mounted on the cyclic triaxial apparatus. Based on the experimental results, the modulus reduction and damping ratio curves are developed for the sand at different confining stresses. The developed modulus reduction and damping ratio curves can be used straightway for carrying out free-field ground response analysis of various project sites located in the south-east coast of India.

## Experimental Investigations

The laboratory investigation programme consisted of two sets of tests. The first set of experiments is composed of the strain-controlled cyclic triaxial tests on the reconstituted sand specimens prepared at initial relative densities of 65 % and 85 % with confining stresses of 100, 150 and 200 kPa according to ASTM D3999-91 (2003). All the specimens are subjected to a cyclic sinusoidal deformation having a frequency of 1 Hz. In the second set of experiments, bender element tests are used for the estimation of maximum shear modulus ($G_{max}$). The bender element tests are carried out on the reconstituted sand specimens prepared at initial relative densities of 65 % and 85 % for effective confining stresses varying from 20 kPa to 200 kPa.

### Cyclic Triaxial Test Set-up

The experimental studies are conducted using Wykeham Ferrance International, UK make, servo-controlled cyclic triaxial testing facility available in the Department of Civil Engineering, Indian Institute of Technology Madras, Chennai. The equipment is completely automated and computerized. It consists of a servo-controlled pneumatic actuator having a capacity of ± 25kN and frequency up to 70 Hz. The load frame is attached to a submersible load cell of capacity ± 5 kN. The triaxial cell has the facility to conduct the tests on soil samples of sizes 50 mm, 70 mm and 100 mm diameters with confining stresses up to 1000 kPa using the pneumatic control panel. Both stress-controlled and strain-controlled tests can be performed using built-in sine, triangular and square waveforms or any other desired loading waveform including earthquake excitations by means of external input. The axial deformation, lateral deformation, volume change, cell pressure, cyclic load and sample porewater pressure can be monitored using a dedicated data acquisition system.

The piezoelectric bender elements are mounted on the top and bottom platens of the cyclic triaxial cell and the bender elements penetrate about 3 mm into the sample. The bender elements are piezoelectric cantilever shaped transducers for generating and detecting shear wave motions through the soil medium. In the bender element test, a pair of bender elements is used whereby one of the bender elements acts as the shear wave transmitter and the other bender element acts as the receiver. The function generator generates an input signal with user defined amplitude and frequency. An oscilloscope is used for recording data from the bender elements. The data of the input and output signals from the oscilloscope are displayed and recorded in the computer using the software. The travel time of the shear wave from the transmitter to the receiver is determined from the wave trace displayed by the software that allows the user to quickly and easily calculate the shear wave velocity.

## Sample Preparation

The physical properties of the sand are determined by conducting the laboratory tests and are summarized in Table 1. A typical grain size distribution of the sand samples used in the study is shown in Figure 1. The samples are characterized as uniformly graded sand (SP) as per the Indian Soil Classification System (IS: 1498-1970).

**Table 1 Properties of Sand**

| ***Property*** | ***Value*** |
|---|---|
| Specific gravity, $G_s$ | 2.68 |
| Maximum dry density (kg /m$^3$) | 1720 |
| Minimum dry density (kg /m$^3$) | 1510 |
| Maximum void ratio, $e_{max}$ | 0.78 |
| Minimum void ratio, $e_{min}$ | 0.56 |
| $D_{10}$ (mm) | 0.17 |
| $D_{60}$ (mm) | 0.40 |
| $C_u$ | 2.35 |
| $C_c$ | 0.92 |
| Nonplastic fines | Less than 5% |

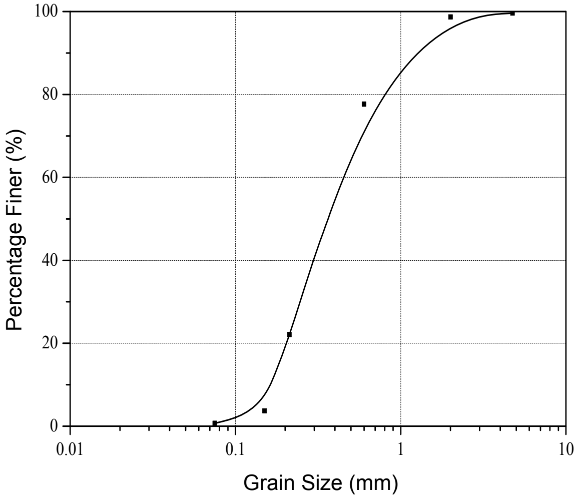


**Fig. 1 Grain Size Distribution of Sand**

Tests are carried out on reconstituted sand samples prepared at the particular relative density by air pluviation technique. Cylindrical soil specimens of 70 mm diameter and 140 mm length are used. After assembling and filling the triaxial chamber with water, de-aired water is allowed slowly to seep through the specimen from the bottom. Back pressure is applied to the specimen in steps and evaluation of the degree of saturation is done at appropriate intervals by measuring Skempton's porewater pressure parameter B and a value of more than 0.96 is ensured. The volume change during saturation is measured using automatic digitally controlled volume change apparatus. Following the saturation, the sand specimens are consolidated to the required effective isotropic stress. The consolidated volume and height of the specimen are obtained using the software through the data acquisition system. The void ratio after consolidation is estimated and found that the post consolidation relative density is only slightly different from the initial relative density. It is due to the fact that the tests are carried out at relatively high densities. The similar observations are also reported in Ishihara (1996). Therefore, in the present study, the test results are interpreted through initial relative density of sand.

The membrane penetration may also affect the volume change of soil specimens. It depends on various factors such as the effective confining stress, grain size, graduation and density of the specimen. However the membrane effect on volume change of the specimen is not considered in the present study, since it is not easy to measure the volume change of specimen due to membrane penetration considering all the above factors. After consolidation the specimens are subjected to a cyclic deviator strain in axial direction.

## Results and Discussion

The dynamic properties of saturated sand such as shear modulus and damping ratio are measured from the cyclic triaxial and bender element tests. The results of the present investigation demonstrate the effects of shear strain, confining stress, number of loading cycles and relative density on the shear modulus and damping ratio.

### Shear modulus and Damping ratio from Cyclic Triaxial Tests

When cyclic triaxial tests are performed on soil specimens, a hysteresis loop will be formed in the plane of measured deviator stress and axial strain for each cycle of loading (N). Figure 2 shows typical stress-strain curves (hysteresis loops) obtained by carrying out a cyclic triaxial test on reconstituted soil specimens at a relative density of 85 % for the effective confining stress of 200 kPa. The slope of the secant line connecting the extreme points on the hysteresis loop is the Young´s modulus (E). The shear modulus (G) is calculated using the expression:

$$G = \frac{E}{2(1+\upsilon)} \qquad (1)$$

The value of Poisson's ratio ($\upsilon$) is assumed as 0.5 for saturated undrained conditions. The damping ratio (ξ) at a given shear strain amplitude (γ) is a measure of dissipated energy versus elastic strain energy and it is computed from area of the hysteresis loop ( $\Delta W$ ) using Equation (2).

$$\xi = \frac{1}{2\pi}\frac{\Delta w}{G\gamma^2} \tag{2}$$

The representative values of shear modulus and damping ratio at a given shear strain amplitude are determined from the hysteresis loop obtained at the tenth cycle of load application, as suggested by Iwasaki et al. (1978) and Kokusho (1980).

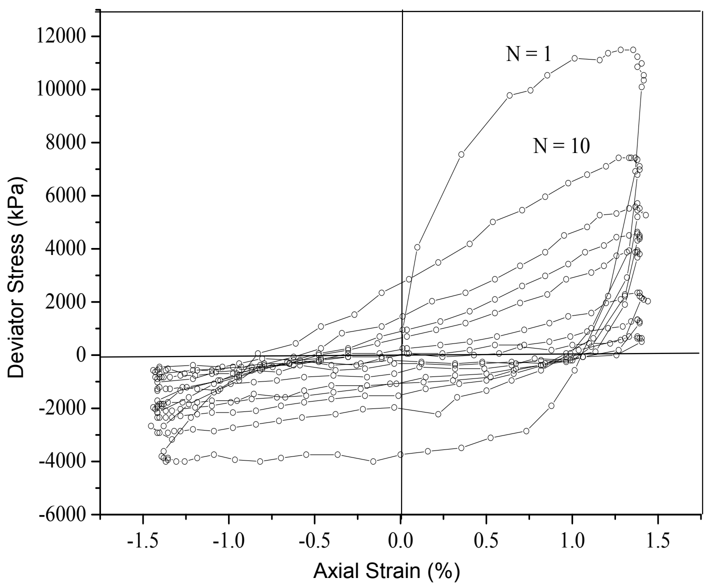


**Fig. 2 Hysteresis Loops of Sand for Different Cycles of Loading, N ($D_r$ = 85 %, $\sigma'_0$ = 200 kPa)**

## Effect of Number of Loading Cycles on Shear Modulus

Figure 3 shows the measured shear modulus with progressive number of cycles of loading at different shear strain amplitudes. The shear modulus decreased with progression of number of cycles up to about 10-15 cycles and after that the effect of number of cycles practically disappears. The G values at 2$^{nd}$ and 10$^{th}$ cycles differ at most by 20 % when shear strain is larger than $10^{-2}$ %. It is found that this degradation is mainly because of the decrease in mean effective stress as the stress paths move toward the origin due to the progressive buildup of porewater pressure as the number of cycles of loading increases. The pore pressure ratios measured with number of loading cycles at various strain levels are plotted in Figure 4.

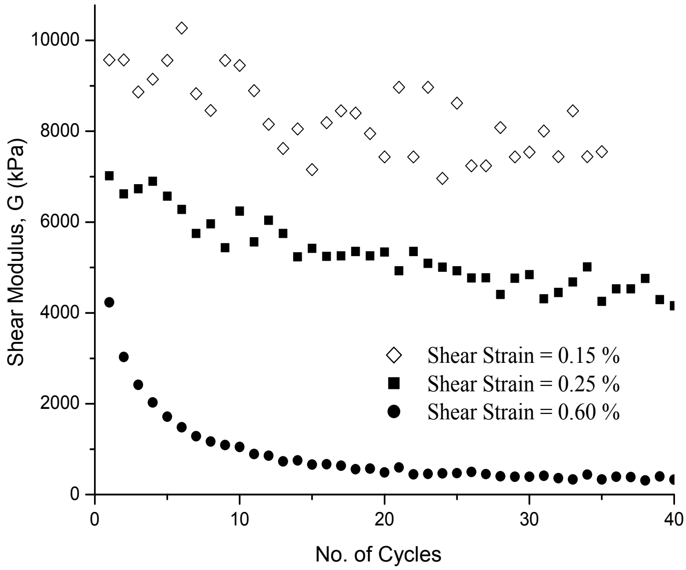


**Fig. 3 Variation of Shear Modulus with Number of Cycles**

**($D_r$ = 65 %, $\sigma_0^{'}$ = 200 kPa)**

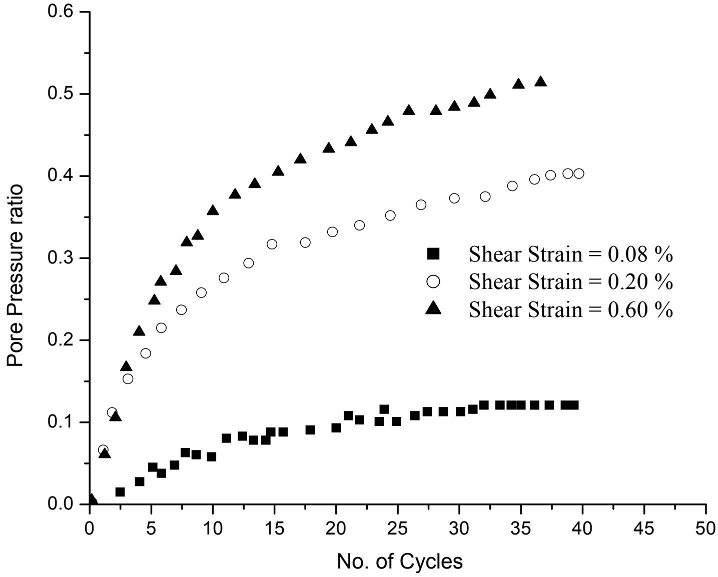


**Fig. 4 Variation of Pore Pressure Ratio with Number of Cycles**

**($D_r$ = 65 %, $\sigma_0^{'}$ = 200 kPa)**

The pore pressure ratio ($r_u$) is defined as the ratio of excess porewater pressure developed during one cycle and the initial effective confining stress ($\sigma_0^{'}$).The magnitude of porewater pressure is increased significantly with increasing number of loading cycles especially at higher strain levels. The rate of increase of pore pressure ratio is rapid in the beginning 10-15 cycles and after that rate of increase is rather slow. This is because in denser sands the number of cycles required to cause particle rearrangement are obviously less. The degradation of fully saturated sands, cyclically loaded in undrained conditions, takes place due to the porewater pressure buildup, which causes the reduction of effective stress and consequently the shear strength and moduli. It is to be noted that the magnitude of porewater pressure buildup and the variation of shear modulus are functions of shear strain amplitude and number of loading cycles.

**Effects of Shear Strain and Confining Stress on Shear Modulus**

The variation of shear modulus of the tested sand with the magnitude of shear strain for different confining stresses is shown in Figure 5. It can be easily noticed from the figure that the shear modulus of the sand consolidated at a particular confining stress decreases drastically with increase in shear strain. At the shear strain range of 0.01 – 1 %, the degradation of stiffness i.e. the reduction of shear modulus with strain occurs rapidly and reaches nearly one tenth of its initial value at a strain level of 1 %. It can also be found from Figure 5 that the shear modulus versus shear strain curves shift consistently to the right along the shear strain axis as the confining stress becomes higher. This observation can be explained by considering the dependency of shear resistance on the confining stress. The observation made above is in concordance with the observations made by Ishibashi and Zhang (1993) and Stokoe et al. (1999).

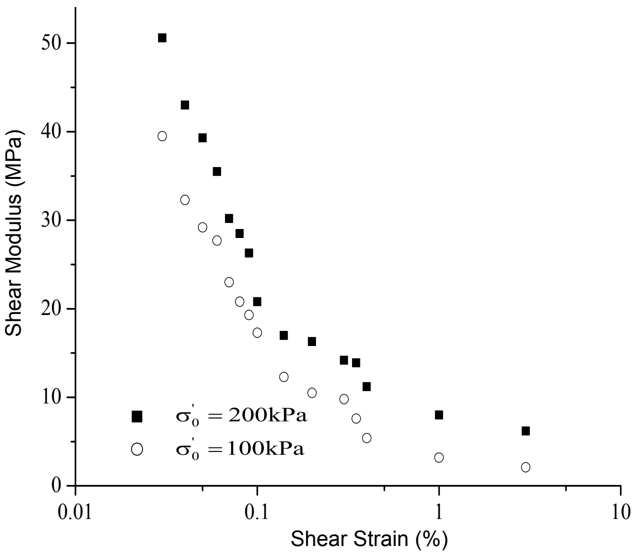


**Fig. 5 Shear Moduli from Cyclic Triaxial Tests ($D_r$ = 65 %)**

## Effect of Relative Density on Shear Modulus

The effect of relative density on the shear modulus is examined by conducting cyclic triaxial tests with two different relative densities. Figure 6 shows the variation of shear modulus with increase in shear strain at different relative densities for an effective confining stress of 200 kPa. The shear modulus increases with increase in relative density in the range of shear strains used for testing.

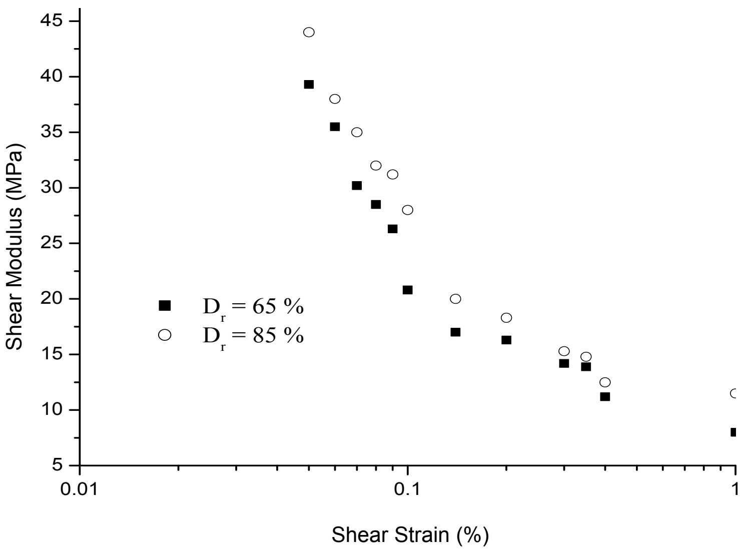


**Fig. 6 Variation of G with Shear Strain at Different Relative Densities**
**($\sigma'_0$ = 200 kPa)**

The modulus ratio ($G/G_{max}$) of the sand with different relative densities is shown in Figure 7. This figure shows practically the same shape of modulus reduction curves for samples with different relative densities due to the fact that both the G and $G_{max}$ are functions of void ratio. The similar behaviour was also observed by Kokusho (1980) for Toyoura sand.

## Effects of Shear Strain and Confining Stress on Damping Ratio

Figure 8 shows the variation of damping ratio ($\xi$) with log $\gamma$ for the reconstituted sand samples with a relative density of 65 %. It is seen from the damping curves that the damping ratio increases with an increase in shear strain amplitude, especially for the shear strain amplitudes larger than $10^{-2}$ %. Damping is an index, representing the amount of energy dissipated during cyclic loading. As the soil particles slide upon adjacent particles under cyclic loading, the strain energy released during the unloading stage is less than the strain energy accumulated during the loading stage. Therefore, at higher strain levels, more slippage and rearrangement of particles occur and higher damping ratio results.

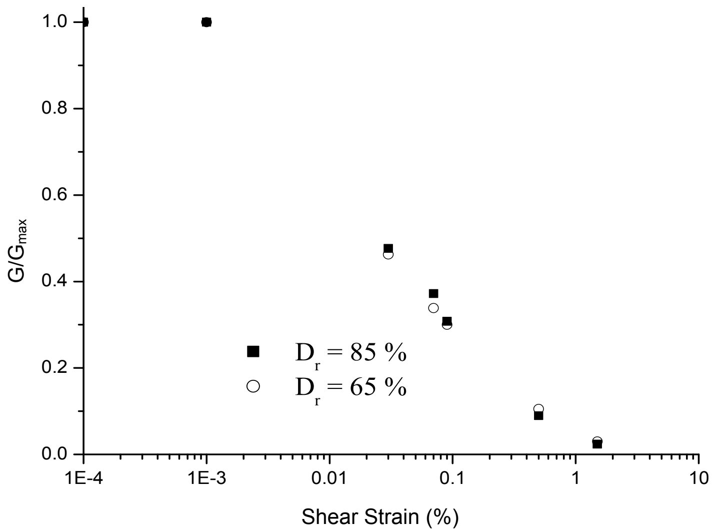


**Fig. 7 Variation of G/G$_{max}$ with Shear Strain at Different Relative Densities ($\sigma_0'$ = 200 kPa)**

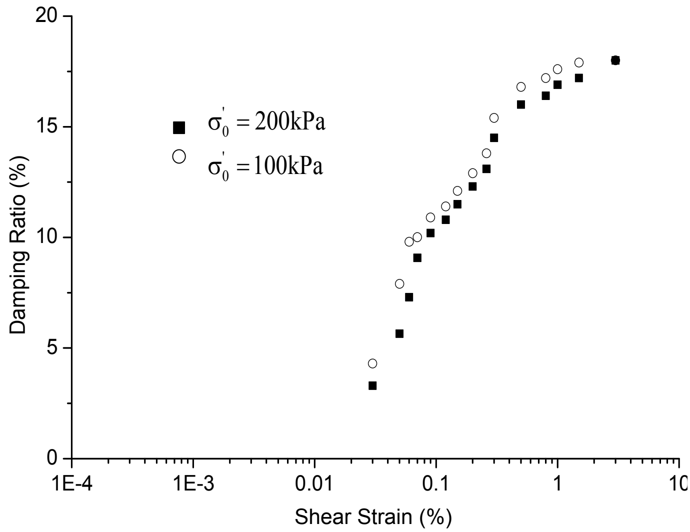


**Fig. 8 Damping Ratio from Cyclic Triaxial Tests (D$_r$ = 65 %)**

It is also noted from Figure 8 that the damping decreases with increasing confining stress for all the values of shear strain amplitudes used for the testing. When the confining stress increases, there are more contacts between the adjacent soil particles in a soil sample, thus there are more pathways for the waves to propagate thereby leading to lesser energy dissipation.

**Effect of Number of Loading Cycles on Damping Ratio**

Figure 9 shows the variation of damping ratio with shear strain amplitude during the progression of number of loading cycles. It is noted that the damping ratio obtained at 2$^{nd}$ and 10$^{th}$ cycles differ nearly by 3% within the strain range of 0.05 – 1%.

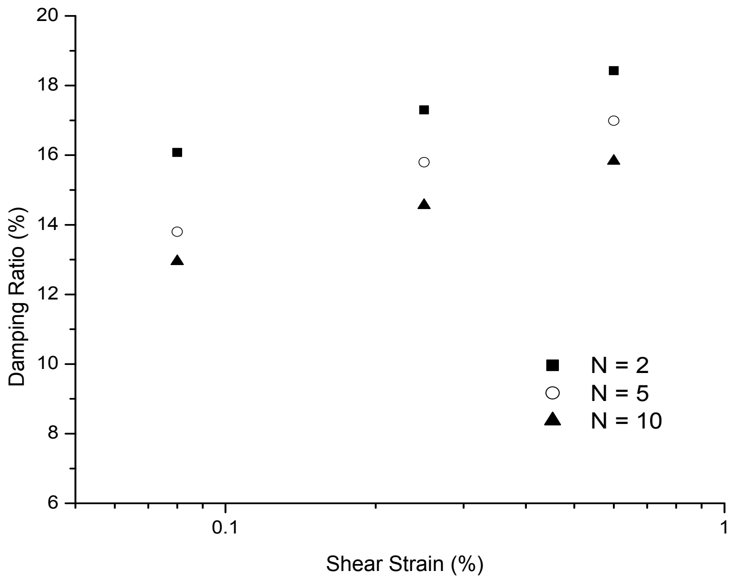


**Fig. 9 Variation of Damping Ratio with Shear Strain at Different Number of Loading Cycles ( $\sigma_0'$ = 200 kPa)**

**Effect of Relative Density on Damping Ratio**

The damping curves obtained for sand samples tested with two relative densities are shown in Figure 10. From the figure it is seen that they are practically same indicating no effect of relative density (65 % to 85 %) on damping at various strain levels. Similar conclusions were also arrived on the effect of relative density on the damping behaviour of sand by Hardin and Drnevich (1972) and Tatsuoka et al. (1978) based on the results of resonant column and torsional shear tests.

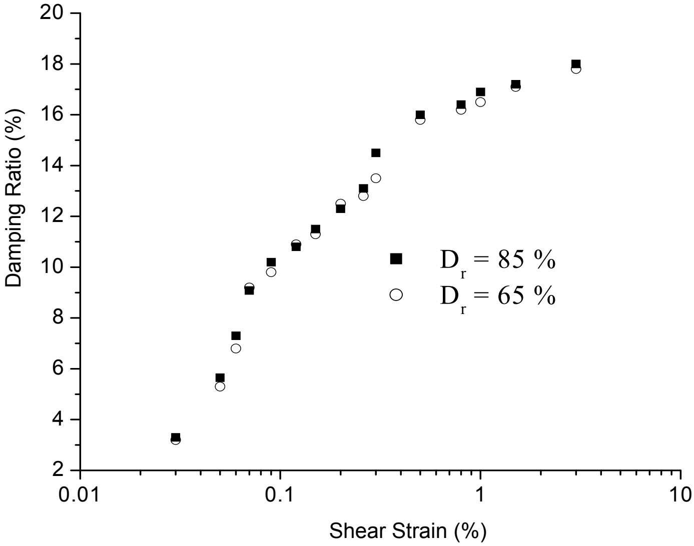


**Fig. 10 Variation of Damping Ratio with Shear Strain for Different Relative Densities ($\sigma_0'$ = 200 kPa)**

## Estimation of Maximum Shear Modulus Using Bender Elements

The maximum shear modulus ($G_{max}$) for saturated sand in the present study is estimated from the shear wave velocity measurements using bender elements. The procedure for the selection of input parameters and the methods of signal interpretation are described by Jaya et al. (2007). The wave traces obtained by the bender element tests are used for the interpretation of the travel time of the shear waves through the sample. The shear wave velocity and hence the $G_{max}$ values corresponding to each effective confining stress are estimated based on the elastic theory of wave propagation as

$$V_s = \frac{L}{t} \tag{3}$$

where $V_s$ = shear wave velocity, L = tip to tip distance between the bender elements, and t = travel time of shear waves through the sample. The $G_{max}$ is related through shear wave velocity as

$$G_{max} = \rho V_s^2 \tag{4}$$

where ρ = mass density of the soil sample.

The $G_{max}$ at different void ratios is estimated under fully saturated conditions for isotropically consolidated samples. The variation of estimated $G_{max}$ value with mean effective confining stress is plotted in Figure 11. The modulus ratio ($G/G_{max}$) is calculated by normalizing the shear modulus with the

$G_{max}$ obtained from the bender elements for each of the confining stresses, where the value G has been estimated by cyclic triaxial tests.

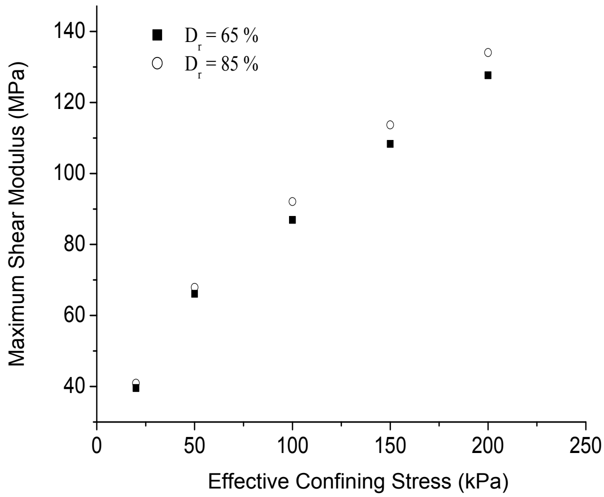


**Fig. 11 Variation of $G_{max}$ with Effective Confining Stress ( $\sigma_0'$ )**

## Predictive Relations for Modulus Reduction and Damping Ratio

The modulus reduction and damping ratio curves are the two important input parameters in the evaluation of nonlinear cyclic behaviour of soil elements as well as in the site response analysis. A variety of nonlinear models are available in the literature in the field of cyclic behaviour of soils. The simplest form of these models is a two parameter model represented by the hyperbolic functions. In the hyperbolic model it is assumed that any stress-strain curve of the soil is bounded by two straight lines which are tangential to it at small strains and at large strains (Hardin and Drnevich 1972). The hyperbolic models have been widely used to describe nonlinear soil behavior under cyclic loading (Stokoe et al. 1999, Zhang et al. 2005, Okur and Ansal 2007).

In the present study, the nonlinear model used to fit the normalized shear modulus from laboratory experiments is a modified version of the hyperbolic model suggested by Stokoe et al. (1999). The following equation is used to describe the relationship of $G/G_{max}$ and log $\gamma$ for the sand specimens:

$$G/G_{max} = 1/\left[1 + \left(\gamma/\gamma_r\right)^a\right] \quad (5)$$

where $\gamma$ = any given shear strain, $\gamma_r$ = reference shear strain, and a = a dimensionless exponent.

The two curve-fitting parameters are the reference strain $\gamma_r$ and the exponent 'a'. The advantage of this equation is that its form is simple and the improved fit to the test data can be obtained by adjusting the curve-fitting parameter 'a' involved in the above equation. The value of the parameter 'a' depends on the soil type. Using the test results, a best fit is obtained for the value of the curve-fitting parameter 'a' as unity.

The reference strain $\gamma_r$ is defined as the value of shear strain where $G/G_{max}$ equals 0.5. This definition of $\gamma_r$ is different from the one proposed by Hardin and Drnevich (1972). The value of $\gamma_r$ is, however, very convenient because it can be determined directly from the laboratory measurements. The reference strain increases with mean effective confining stress as observed from test results of the modulus reduction. Functionally, $\gamma_r$ can be defined as a function of mean effective confining stress as suggested by Stokoe et al. (1995) and the same is used in this study:

$$\gamma_r = \gamma_{r1}(\sigma'_o / p_a)^k \tag{6}$$

where $\gamma_{r1}$ = reference strain at a mean effective confining stress of 100 kPa and k is a stress correction exponent. The value of k is determined as 0.42 by performing regression analysis on the cyclic triaxial test data for different confining stresses.

It is easy to express the variation of damping ratio with cyclic shear strain amplitude as a function of normalized shear modulus, which is less susceptible to possible errors. Hence, Aggour and Zhang (2006) suggested the following expression for the evaluation of damping ratio as a function of $G/G_{max}$:

$$\xi(\%) = Ae^{-c(G/G_{max})^d} \tag{7}$$

where A, c and d are the regression constants.

Based on the cyclic triaxial test results obtained from the present study, it is possible to establish the following relationship by simple regression analysis:

$$\xi(\%) = 20e^{-3(G/G_{max})^{1.2}} \tag{8}$$

At very small strain levels, the value of damping ratio is approximately equal to 1% and at very large strain levels the maximum predicted value of $\xi$ is around 20%. Hence, the proposed Equation (8) appears to be adequate to define the strain dependent damping behaviour of the sand samples considered in the study.

Figures 12 and 13 show the normalized shear modulus reduction and damping curves for different confining stresses obtained from the predictive relations developed [Equations (5) and (7)]. The modulus reduction and damping curves proposed for sands by Seed and Idriss (1970) are also shown in the figures. It can be seen from Figure 12 that the modulus reduction for the confining stress of 400 kPa follows the Seed and Idriss (1970) upper bound curve, while the curve for 100 kPa follows Seed and Idriss (1970) mean curve.

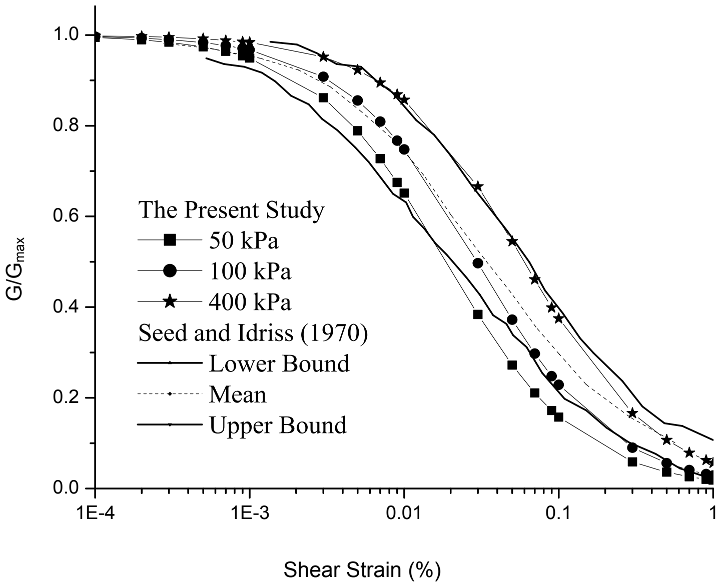


**Fig. 12 Comparison of Modulus Reduction Curves Developed with Curves Proposed by Seed and Idriss (1970)**

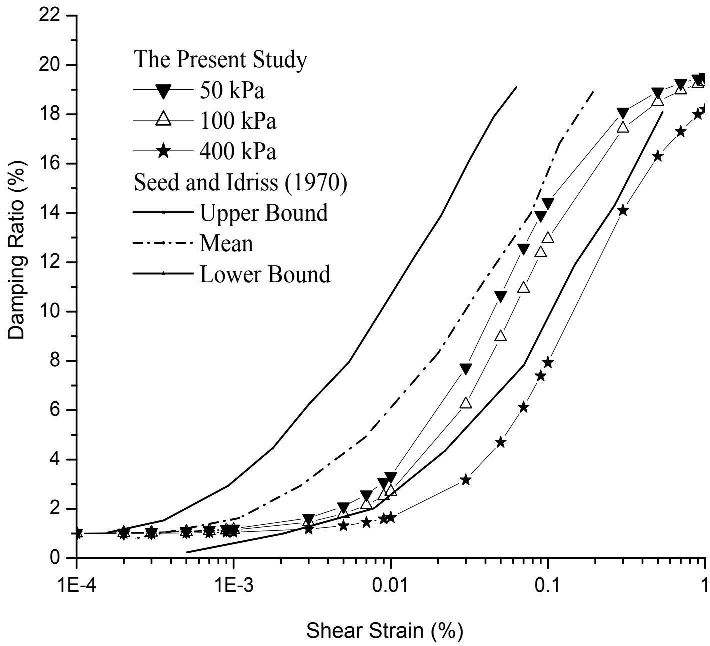


**Fig. 13 Comparison of Damping Ratio Curves Developed with Curves Proposed by Seed and Idriss (1970)**

It can also be observed from Figure 12 that the initial portions of the modulus reduction curves tend to be more flat with an increase of the effective confining stress indicating the linear behaviour of the sand at high confining stress for a wide range of strains.

The effect of confining stress on modulus reduction and damping of sand is found to be significant compared to other factors. As shown in Figure 13, the damping ratio curve for the confining stress of 100 kPa lies between the Seed and Idriss (1970) mean and lower bound curves. But the shape of the damping curve corresponding to the high confining stress as shown in Figure 13 indicates the presence of low hysteretic damping at low to medium strain levels. It is noted from the figures that both the modulus reduction and damping curves shift to the right with the increase in mean effective confining stress. The specific effect of confining stress is not available in the curves proposed by Seed and Idriss (1970). The developed curves take into account the effect of confining stress on modulus reduction and damping ratio and permit proper consideration of the nonlinear characteristics of the sand layers with depth. These curves indicate that, when the effect of confinement is taken into account more appropriately, the degree of nonlinearity of the soil decreases with depth.

## Conclusions

The cyclic behaviour of saturated sands of Kalpakkam region is studied by carrying out undrained strain-controlled cyclic triaxial tests and bender element tests for a wide range of shear strains. A nonlinear model for the modulus degradation of the Kalpakkam sand is proposed based on the modified hyperbolic model. The curve-fitting parameters involved in the modified hyperbolic model are evaluated using regression analysis based on the experimental results. The modulus reduction and damping ratio curves obtained for the Kalpakkam sand are dependent on the mean effective confining stress. The sand specimens tend to exhibit linear behaviour as the effective confining stress is increased from 100 to 400 kPa. It has been found that the proposed modulus reduction curves for low confining stress match well with the lower bound curves and at high confining stress they match well with the upper bound curves of Seed and Idriss (1970). The proposed damping curves for the sand for different confining stresses lie between the mean and lower bound damping curves of Seed and Idriss (1970). The modulus reduction and damping curves developed for Kalpakkam sand can provide guidance on how the sands in the south-east region of India will behave during seismic loading and they can be used within the range of considered confining stresses to carry out site response analysis.

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